RMF Current Drive in FRCs

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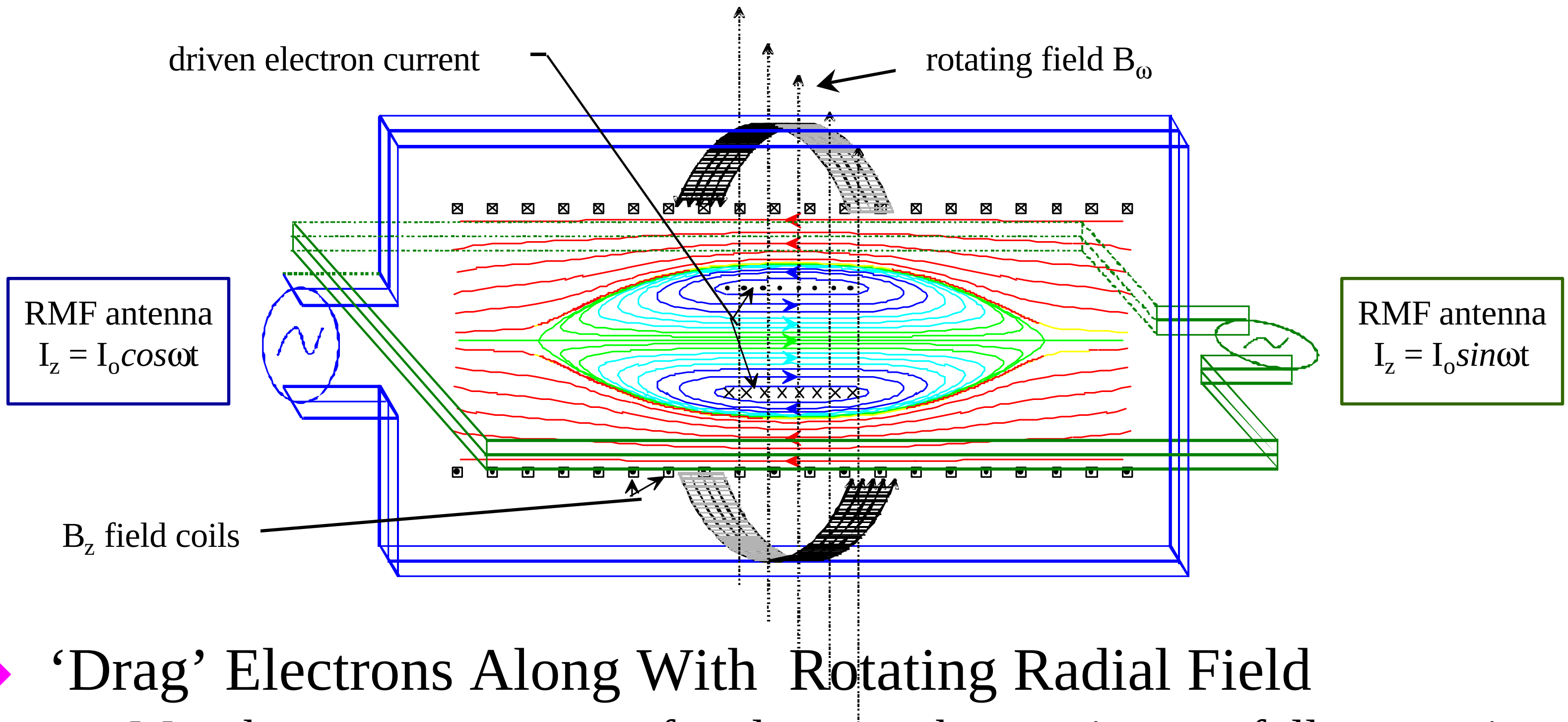
Outline

- Introduction - basic experimental observations
- RMF Drive Consistent With FRC Equilibrium
- STX Experiments
- What It All Means & Continuing Investigations

RMF Current Drive



- 'Drag' Electrons Along With Rotating Radial Field
  - Must have $\omega_{ci} < \omega << \omega_{ce}$ for electrons, but not ions, to follow rotation
- Electrons Magnetized on Rotating Field Lines ($\omega_{ce}\tau >> 1$)
  - Necessary for efficient current drive
  - Absolutely necessary for rotating field penetration

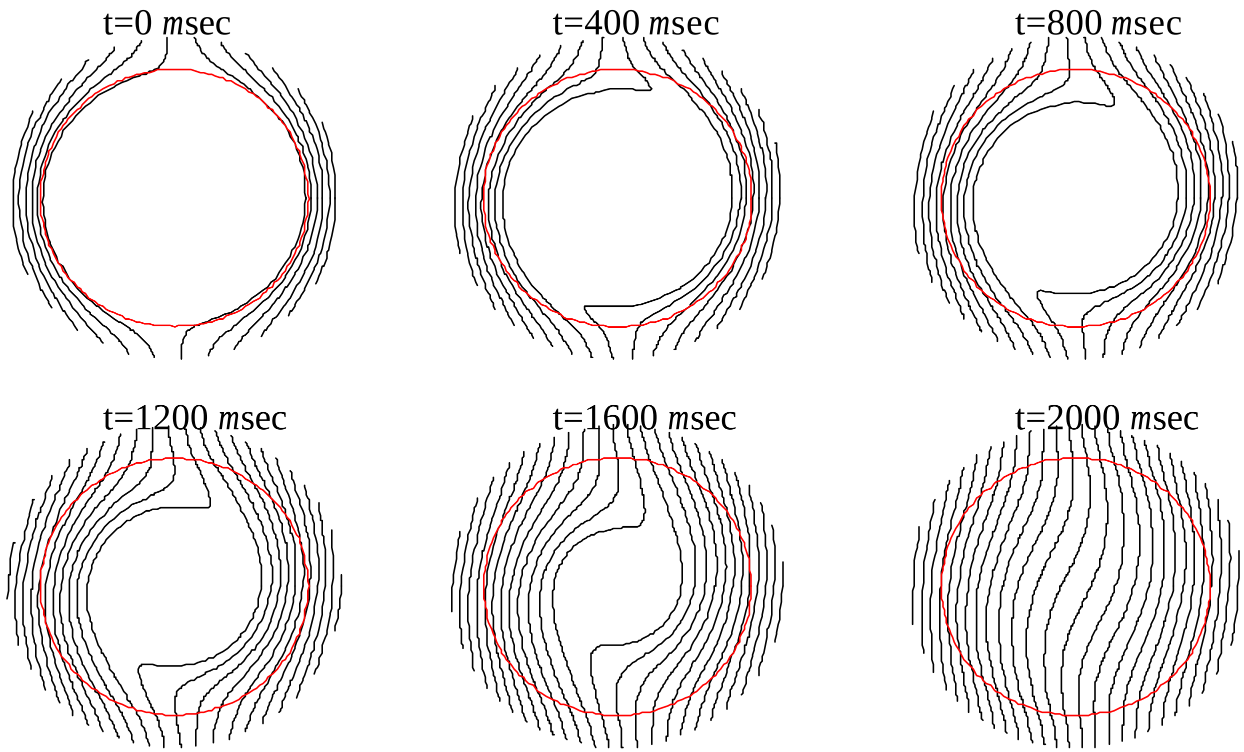
Summary of Basic Physics

- RMF flux build-up and sustainment is made possible by synchronous electron current drive which allows penetration.
- Penetration is possible when RMF force, $2B_\omega^2/\mu_o r$, exceeds resistive drag, $n_e m_e \nu_\perp \omega r$, which we characterize as

$$\gamma = \frac{\omega_{ce}}{\nu_{ei}} > \lambda = \frac{r}{\delta} \qquad \omega_{ce} = \frac{eB_\omega}{m_e} \qquad \delta = \sqrt{\frac{2\eta}{\mu_o \omega}}$$

- If $\gamma > \lambda$ then penetration will proceed just far enough to reverse the external confinement field. Current is sustained on the inner field lines by induced inward flow.
- High FRC $\langle\beta\rangle$ and low separatrix density results in narrow edge current layer. There is a delicate balance between having too few and too many electrons.

RMF Penetration Calculations for Simple Fixed Column



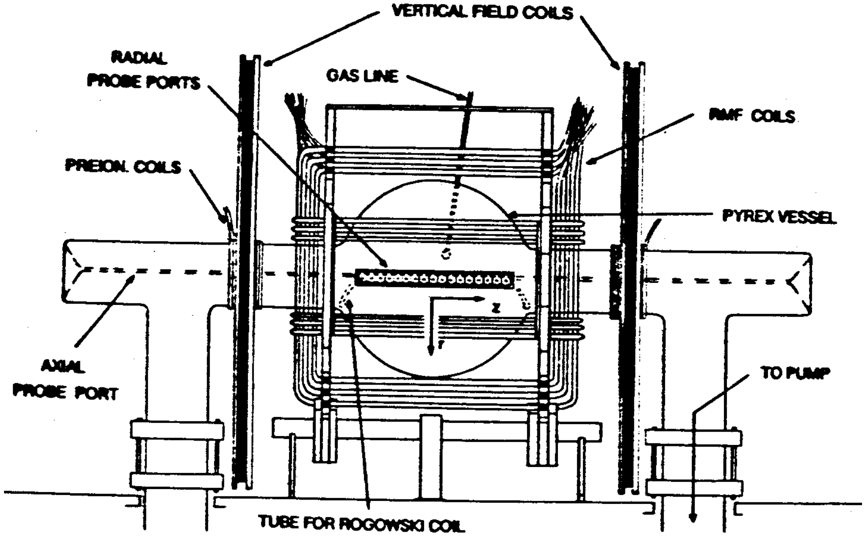
$\lambda = 45$

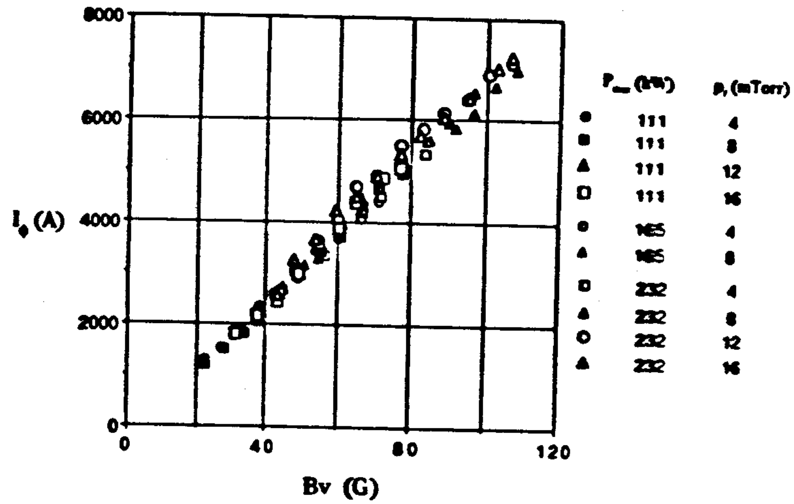
$\gamma = 100$

$r_s = 20$ cm, $n_e = 0.25 \times 10^{14}$ cm^{-3}, $\omega = 10^6$ s^{-1}, $B_\omega = 50$ G

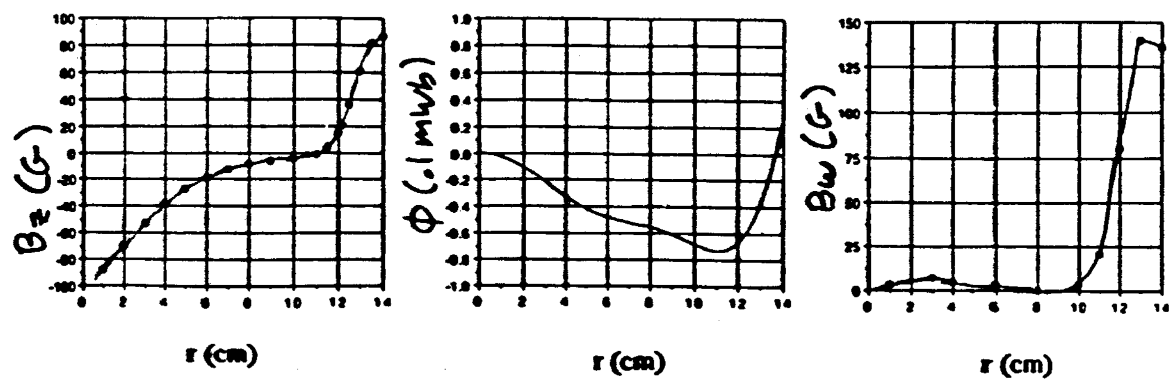
$T_e = 100$ eV, $\eta = 10\ \mu\Omega$-m (10x classical)

Flinders 10ℓ Rotamak

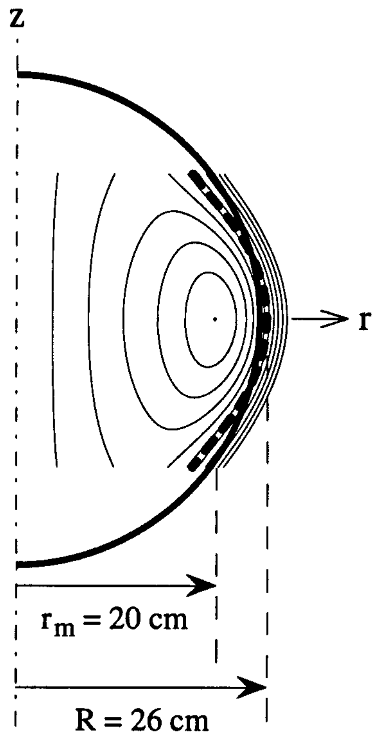


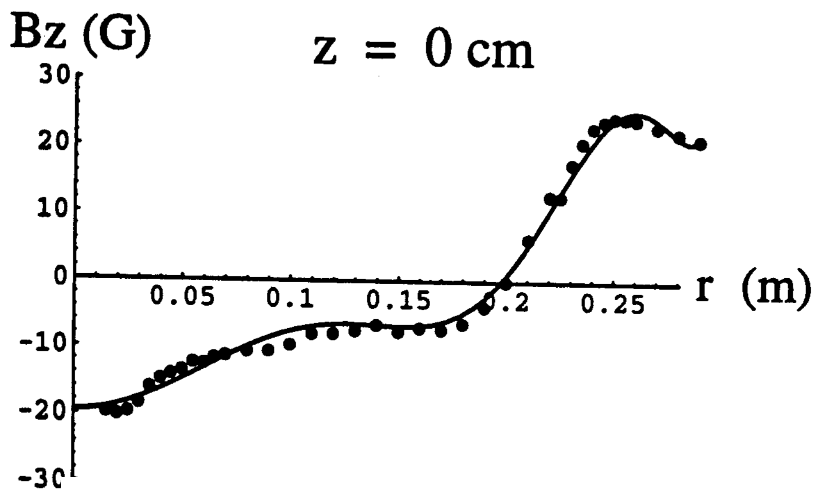


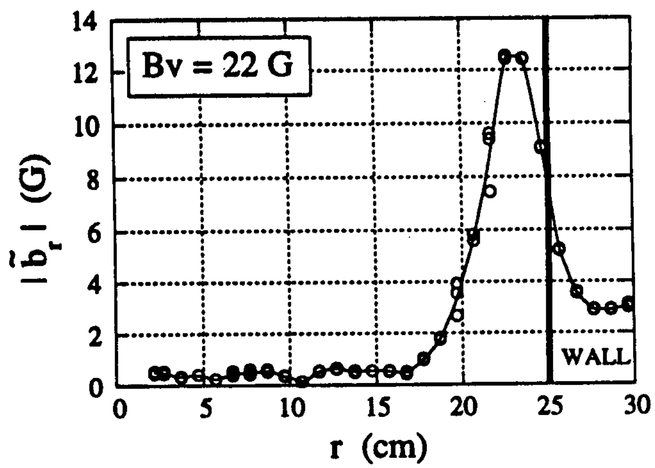
RMF penetration adjusts to provide current necessary to maintain equilibrium



Flinders 50 ℓ Rotamak

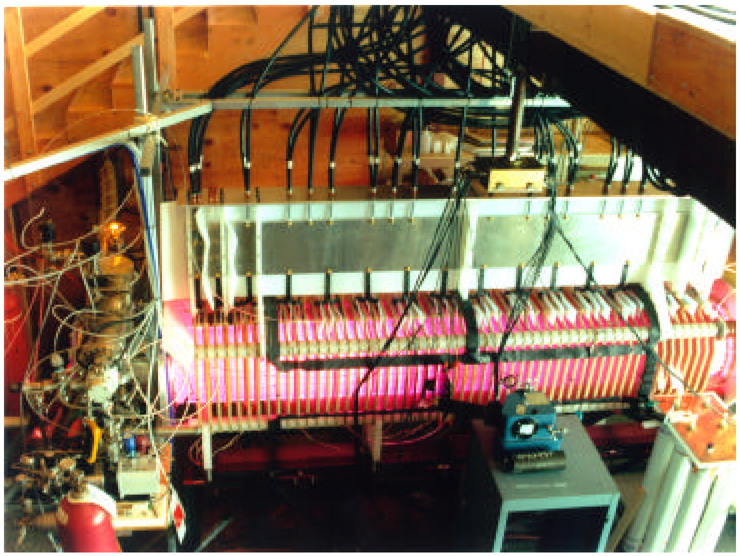


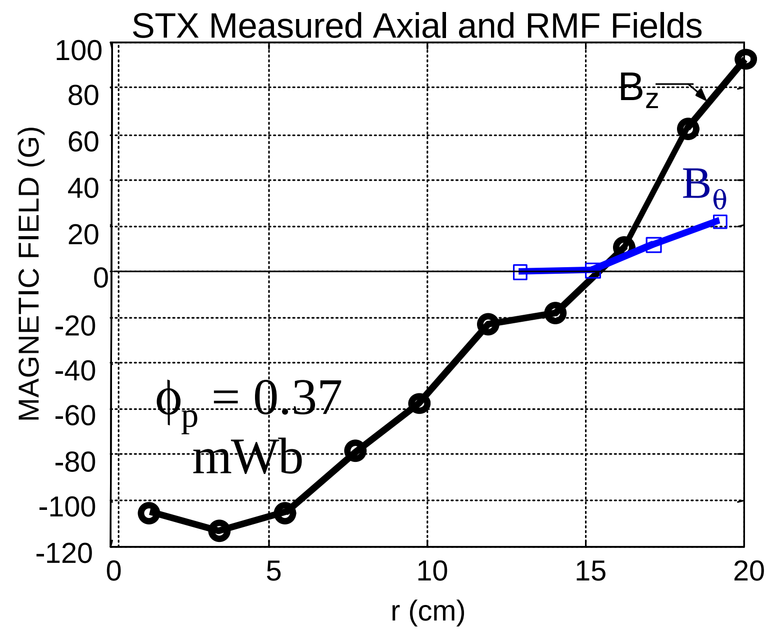




RMF flux drive pushes FRC against plasma tube wall

STX RMF Driven FRC



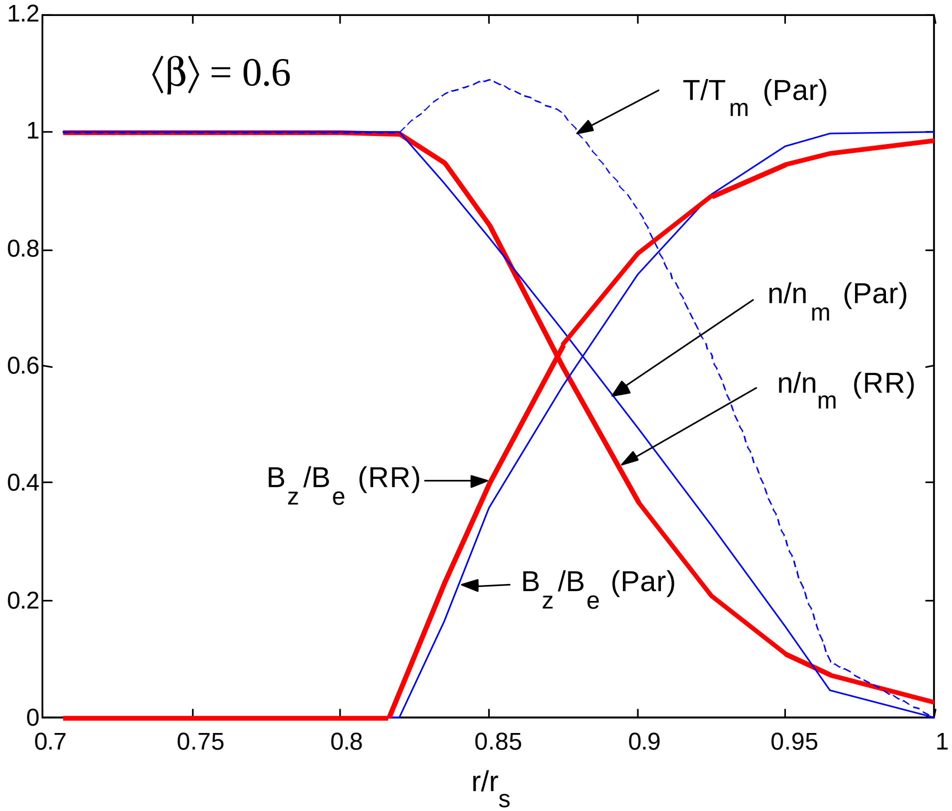


Flux conserver causes external field to increase as FRC expands.

In this experiment separatrix, $r_s \approx 20.7$ cm, is slightly outside plasma tube wall, $r_w = 20$ cm, but density is essentially zero there ($\beta_s \approx 0$).

Internal field exceeds external field due to RMF field contribution.

Density and B_z Profiles Consistent With high $\langle\beta\rangle$, low $n(r_s)$, & $j = ne\omega r$



$$\mathrm{RR:}\ \ n = n_m \mathit{sech}^2 K\left(\frac{r^2}{r_e^2} - 1\right)$$

$$K = \frac{n_m e\omega r_e^2}{2B_e/\mu_o} \qquad T = \mathrm{const}$$

$$\mathrm{Par:}\ \ n = n_m \frac{r_s^2 - r^2}{r_s^2 - r_e^2}$$

RMF Penetration Calculation Including FRC Quasi 2-D Dynamics

Initially

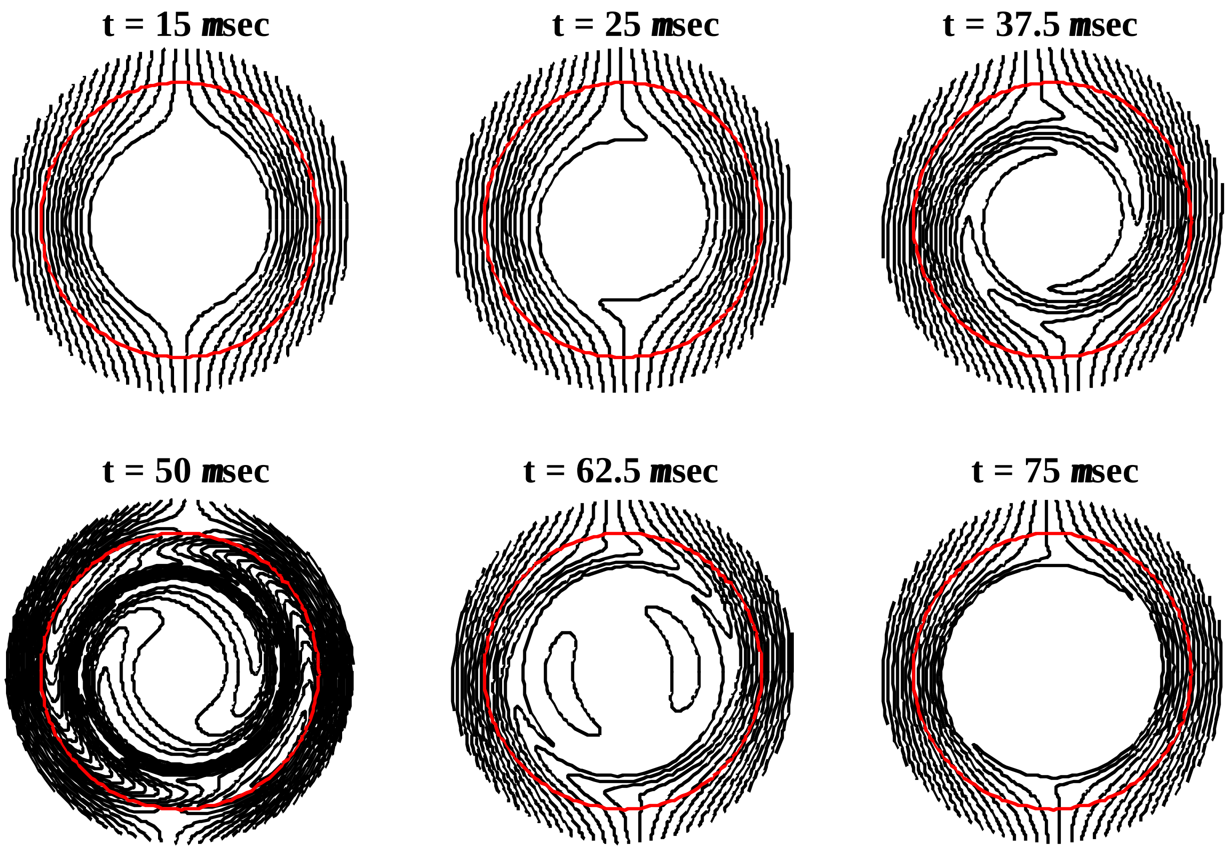
$\lambda = 35$

$\gamma = 155$

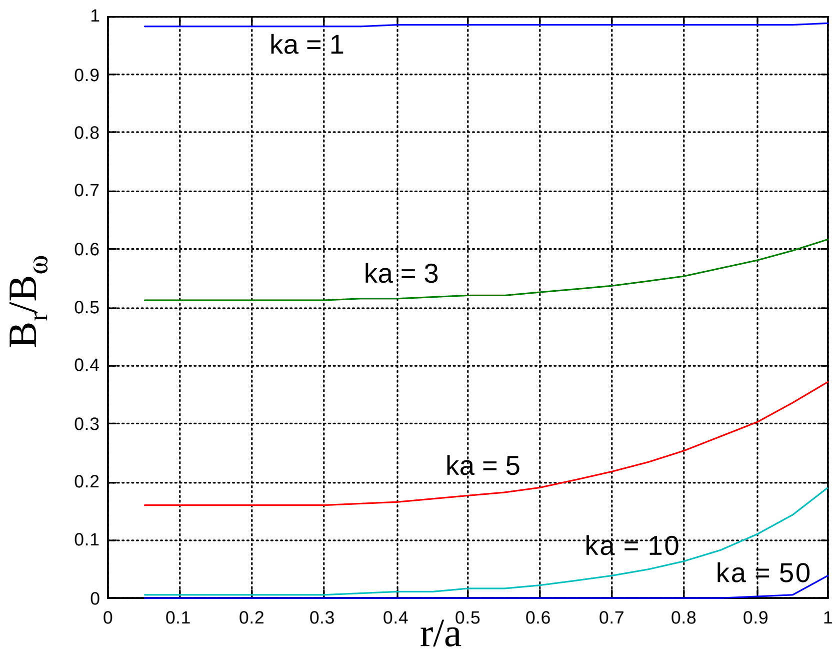
Finally

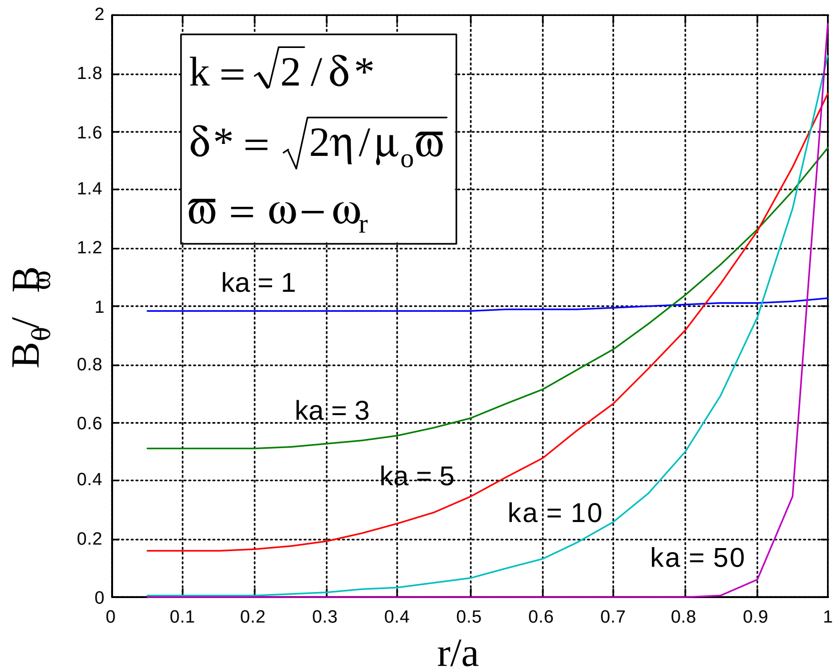
$\lambda = 35$

$\gamma = 47$



Analytic Model Can Give Forces





$$E_z = \omega r B_r \approx \left\{ e^{-i\left(\frac{\pi}{4}+\frac{a-r}{\delta^*}\right)} \right\} \omega \sqrt{\frac{2a}{r}}\, \delta^*\, e^{-\frac{a-r}{\delta^*}} \qquad j_z = \frac{\varpi}{\omega}\frac{E_z}{\eta}$$

$$F_\theta = \langle j_z B_r \rangle = \frac{2B_\omega^2}{\mu_o a}\left(\frac{a}{r}\right)^2 e^{-2\frac{a-r}{\delta^*}}$$

$$T_M \approx \frac{2\pi B_\omega^2}{\mu_o} a\delta^* \qquad (\text{for } \delta^*/a < 0.3)$$

RMF Radial Pressure Gradient

- $F_r = -\langle j_z B_\theta \rangle$
- Analytic solution for edge current layer: $\begin{cases} j_z = \left\{ e^{i\left(\frac{\pi}{4}+\frac{a-r}{\delta^*}\right)} \right\} \dfrac{\sqrt{2}\varpi B_\omega \delta^*}{\eta_{//}} \sqrt{\dfrac{a}{r}} e^{-\left(\frac{a-r}{\delta^*}\right)} \\ B_\theta = \left\{ e^{i\left(\frac{\pi}{2}+\frac{a-r}{\delta^*}\right)} \right\} 2B_\omega \sqrt{\dfrac{a}{r}} e^{-\left(\frac{a-r}{\delta^*}\right)} \end{cases}$
- J_z and B_θ $\pi/4$ out of phase so

$$\langle j_z B_\theta \rangle = \frac{1}{2\sqrt{2}} |j_z| |B_\theta| = \left(\frac{a}{\delta^*}\right) \frac{2B_\omega^2}{\mu_o a} \frac{a}{r} e^{-2\left(\frac{a-r}{\delta^*}\right)}$$

- Resultant radial pressure $p_r = \int_0^a F_r dr = \dfrac{B_\omega^2}{\mu_o}$ is strong.
- This is in addition to $\langle j_z B_r \rangle$ that counters diffusion: $v_r = -\dfrac{1}{B_z}\left(\eta_\perp j_\theta + \langle j_z B_r \rangle / ne\right)$

Average Torque Based Calculation of Flux Build-up

Average $\langle E_\theta \rangle_R \sim \langle F_{\theta M} - F_{\eta M} \rangle / ne$

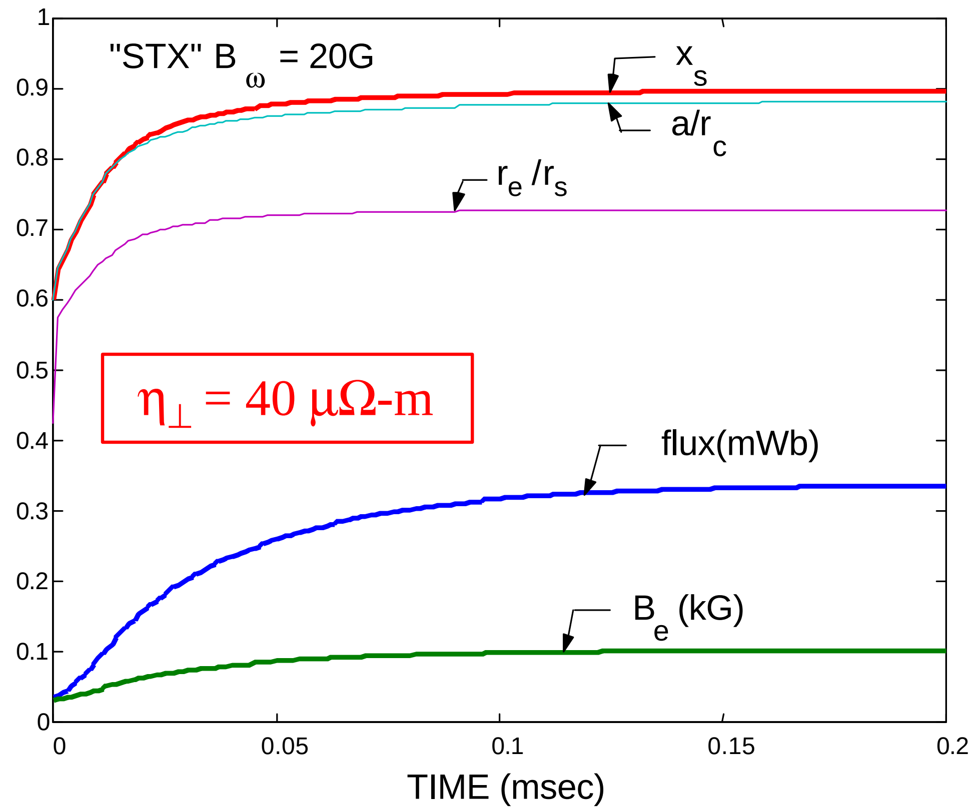
$T_M = T_o(\delta^*/a) \quad T_\eta \approx (\lambda^2/2\gamma^2)T_o$

$T_o = 2\pi a^2 B_\omega^2/\mu_o$

$d\phi/dt = 2(T_M - T_\eta)/nea^2$

$$\boxed{\begin{aligned}\frac{d\phi}{dt} &\sim \frac{T_o}{nea^2} = \frac{2\pi B_\omega^2}{ne} \\ &= 0.004 \frac{B_\omega^2(G)}{n\left(10^{20}\,m^{-3}\right)} \frac{mWb}{m\,sec}\end{aligned}}$$

Flux build-up continues until λ ~> γ (due to field compression and density increase). Results in large x_s.

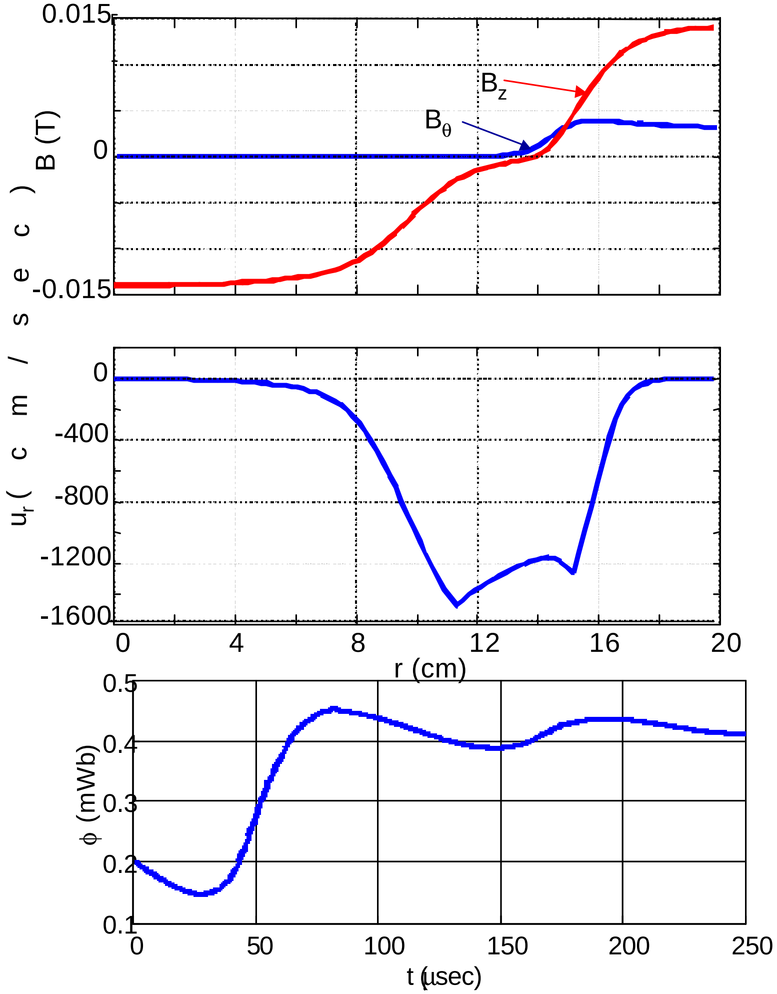


Details of Steady Solution

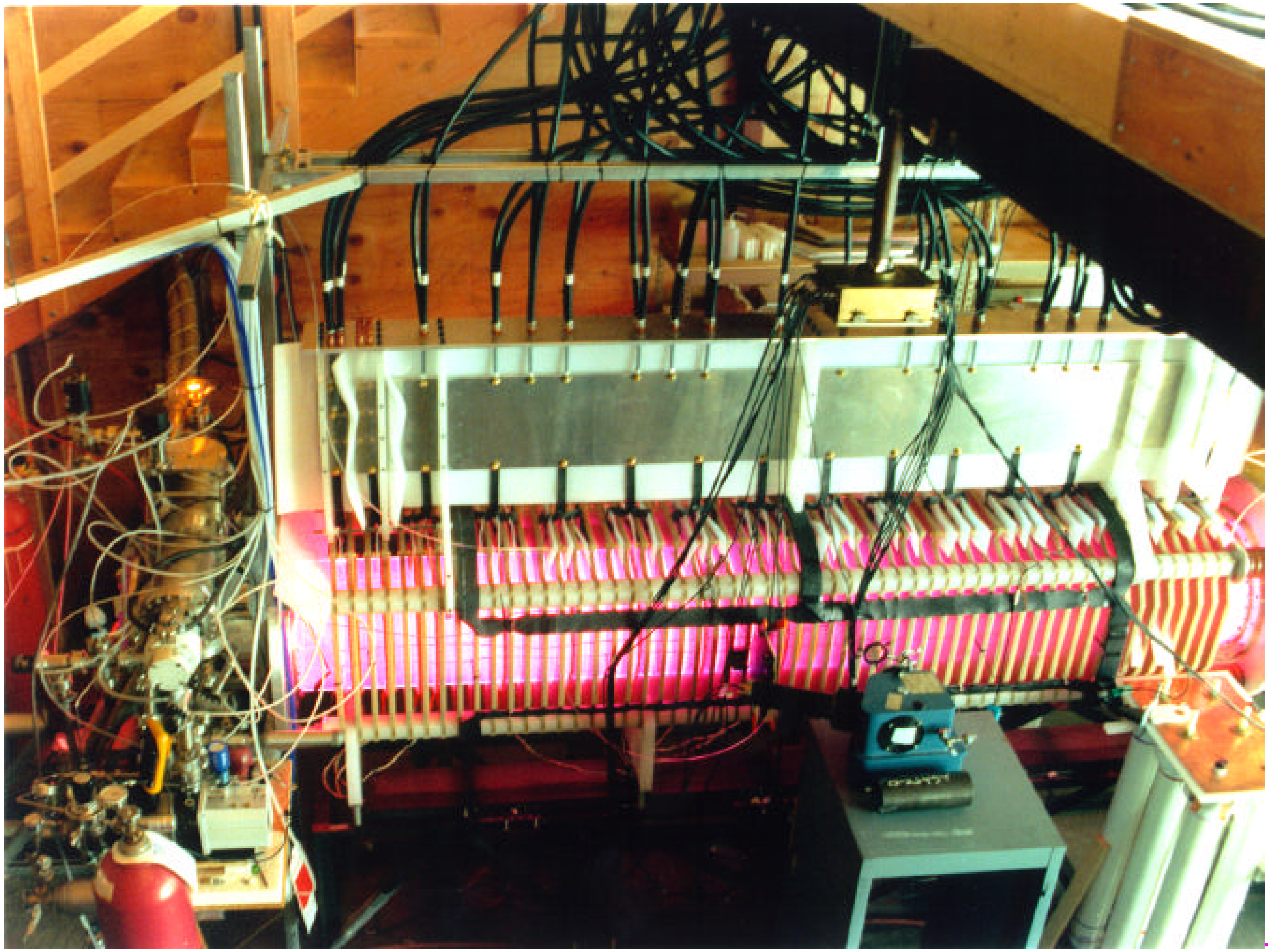
- True steady state requires $E_\theta = 0$ everywhere.

$$E_\theta = \eta_\perp j_\theta + v_{er} B_z - \langle v_{ez} B_r \rangle$$

- 'Quasi-2D' numerical solution shows how this can occur due to overall inward flow, with RMF current drive only in outer edge.
- Calculation duplicates measured $B_z(r)$ profile.
- Numerical flux build-up rate ≈ simple analytic rate for stipulated $\eta_\perp$ = 40 μΩ-m.

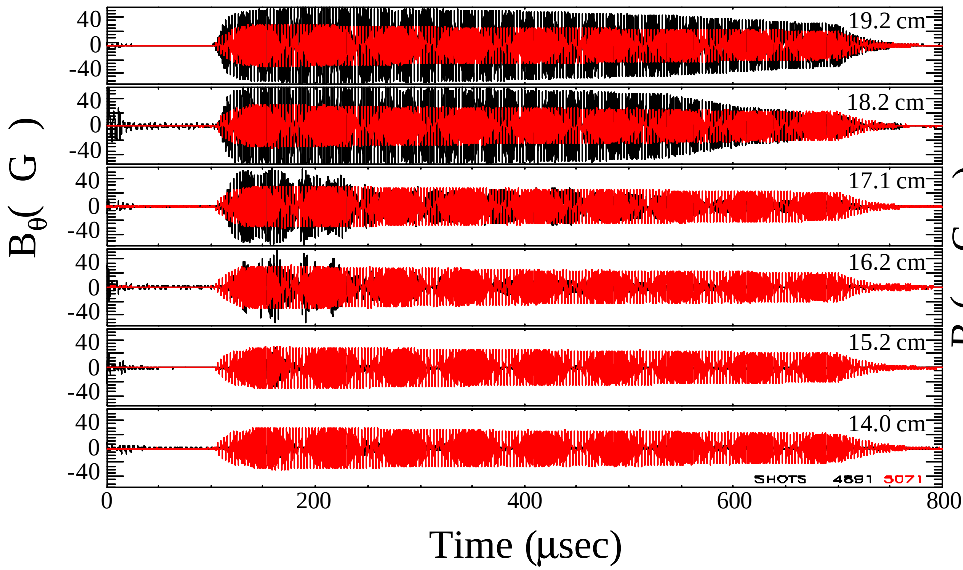


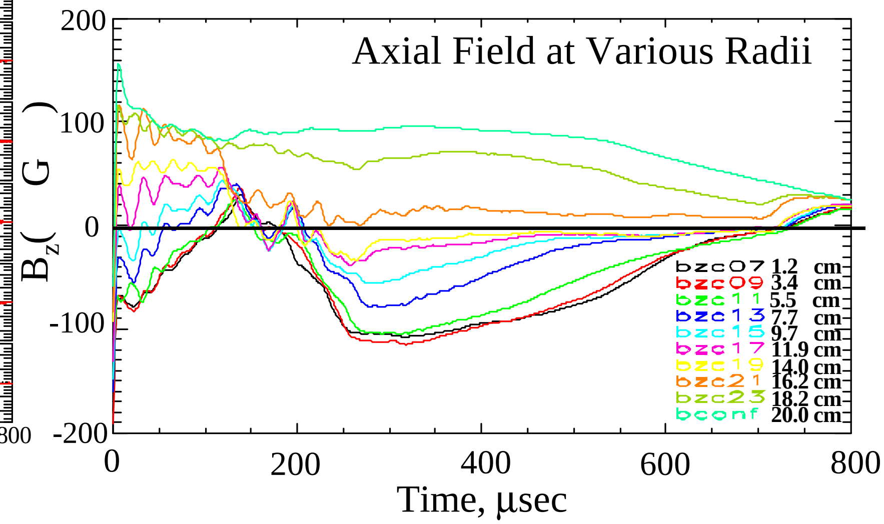
STX Experiments



Rapid Flux Build-up From Fully Ionized (decayed FRC) Plasma Column

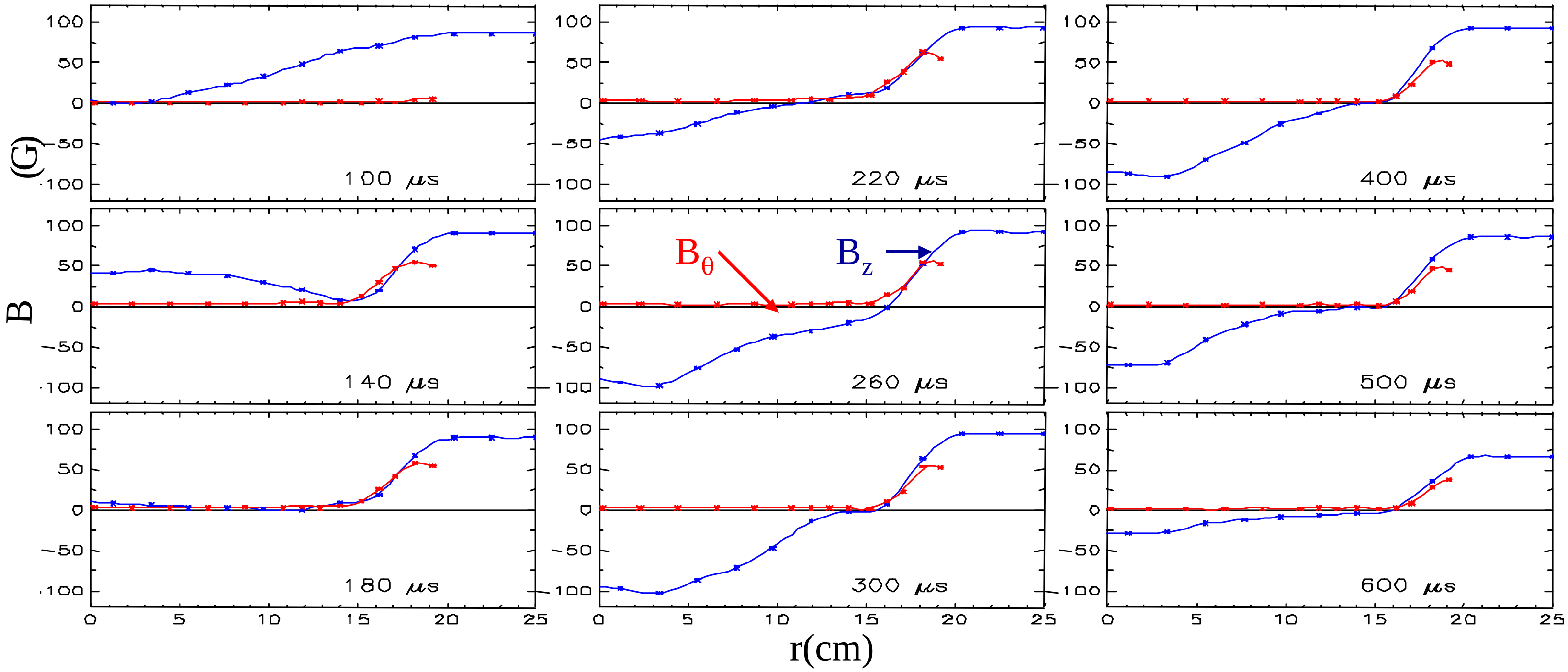
Red - vacuum shot
Black - with plasma





- Flux builds up to 0.37 mWb in 100 μsec in agreement with calculations.
- Flux then decays slowly: most likely due to overheating and too low a density to produce current reversal in equilibrium edge layer, or to inability to sustain inward v_r throughout column due to 2-D effects.
- Ion spin-up could also reduce maximum synchronous current, but not seen from Doppler broadening measurements

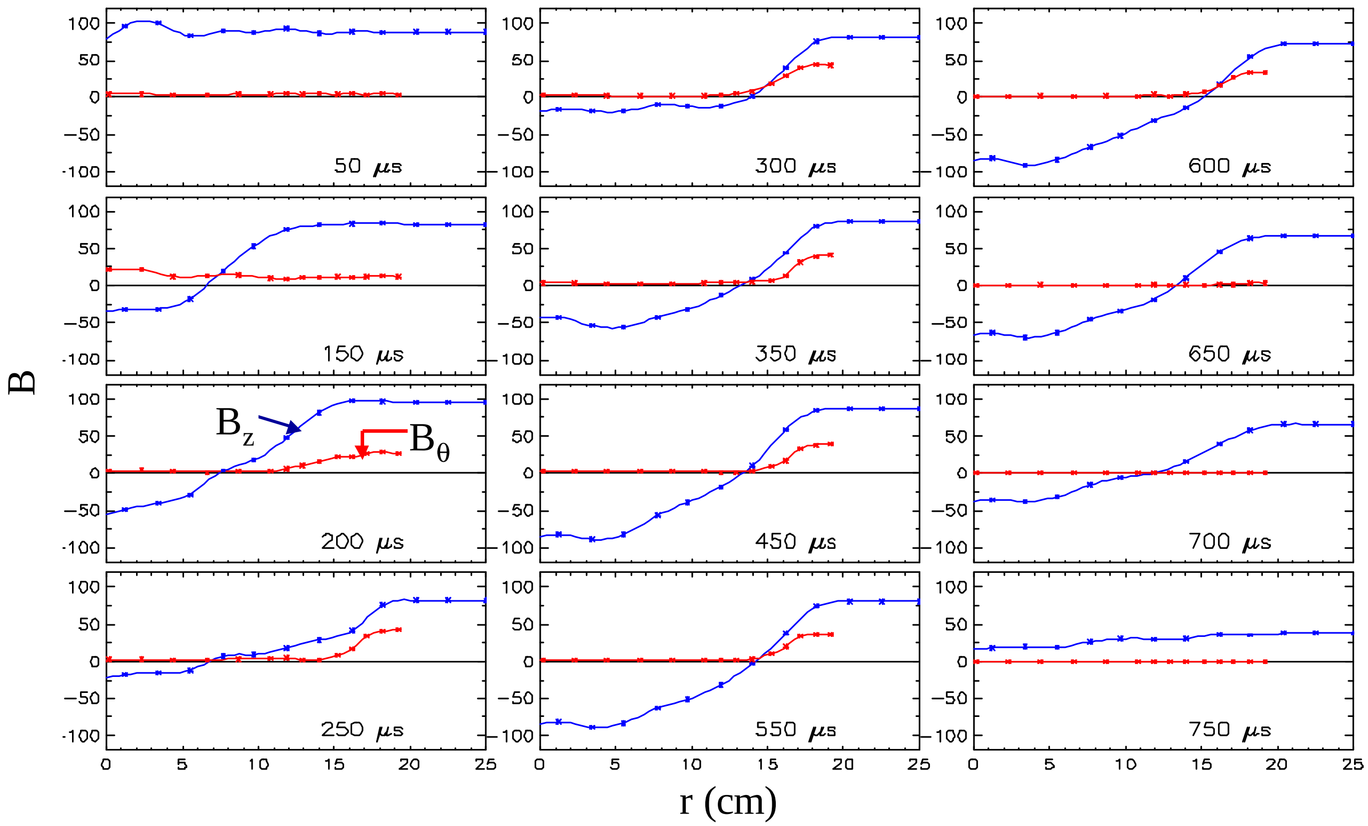
Details of Rapid Flux Build-up Case



- FRC transitions to high beta column. This is only possible steady solution if total line density is too low to maintain $I_o' = \langle n_e \rangle e \omega a^2/2 > 2B_e/\mu_o$
- $\langle n_e \rangle = 2.5\text{x}10^{18}$, $\omega = 2.2\text{x}10^6$, $a = 0.2$, $B_e = 0.009$:

  $I_o' = 18$ kA/m, $2B_e/\mu_o = 14$ kA/m

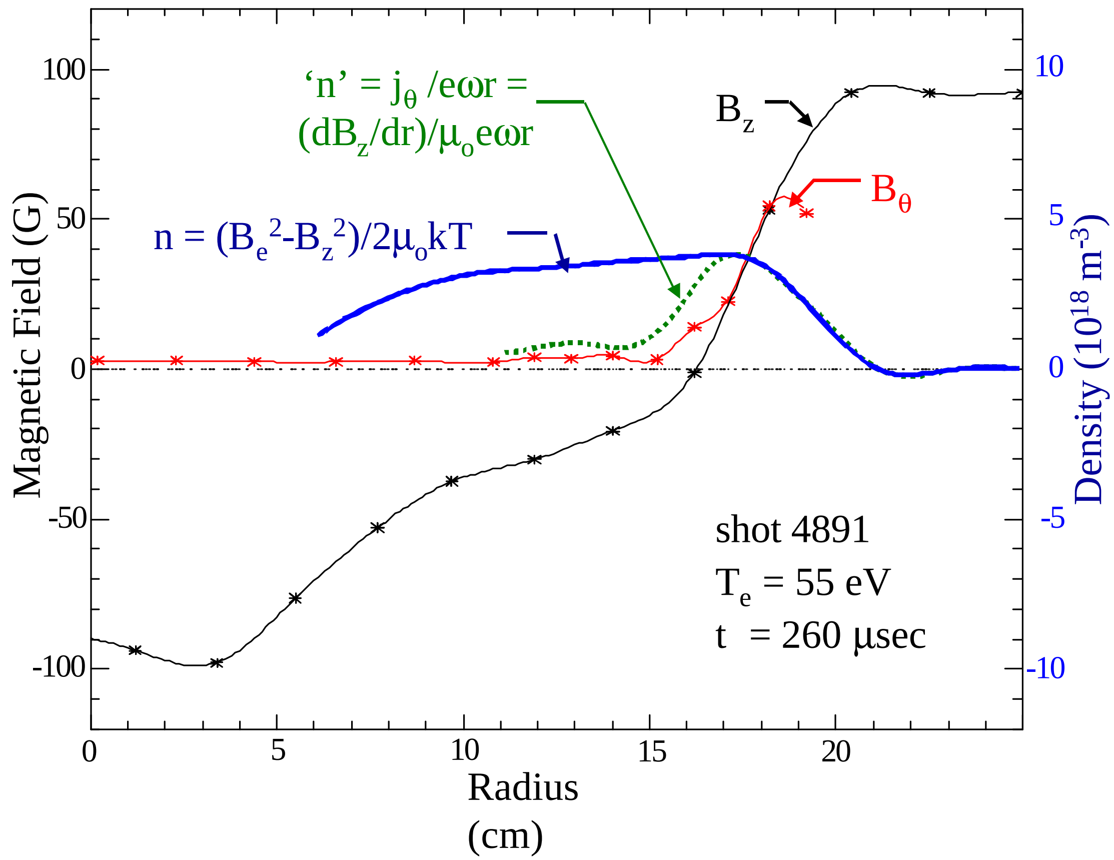
Flux Build-up Starting From Low Beta Column



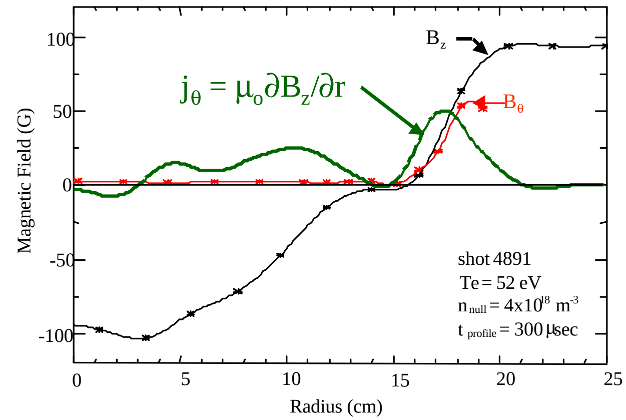
- Steady state achievable under different operating conditions.
- Flux decay rate after RMF turnoff $\Rightarrow \eta_\perp \sim 40\ \mu\Omega$-m.

Electron rotation appears synchronous in driven edge

- 'n' calculated from synchronous electron rotation agrees with n inferred from pressure balance assuming fixed 'best fit' temperature over region of RMF penetration.
- Very low density at r = 0 and separatrix (r_s ~ 20 cm).
- RMF field contributes to confinement with $B_e(0) > B_e(r_s)$



Estimate of STX Resistivity



- Numerical calculations match measured flux build-up using $\eta_\perp = 40\ \mu\Omega$-m.
- Flux lifetime without RMF drive $\tau_\phi = r_s^2/16(\eta_\perp\mu_o) \sim 80\ \mu s \Rightarrow \eta_\perp = 40\ \mu\Omega$-m.
- Implied absorbed Ohmic power = $\eta_\perp \int j^2 dV \sim 3.5\ \eta_\perp(\mu\Omega\text{-m})$ kW $\Rightarrow$ 140 kW.
- $E_p = 1.5NkT_e = 8$ J would yield $\tau_E = 57\ \mu s$.
- STX RMF power supply is 1.5 kJ and decays ~500J in 0.5 ms; ~ 1000kW with and without plasma. Best estimate is extra plasma absorbed power ~60kW. This implies lower $\eta_\perp$ where current flows. Better measurements of absorbed Ohmic power are critical.

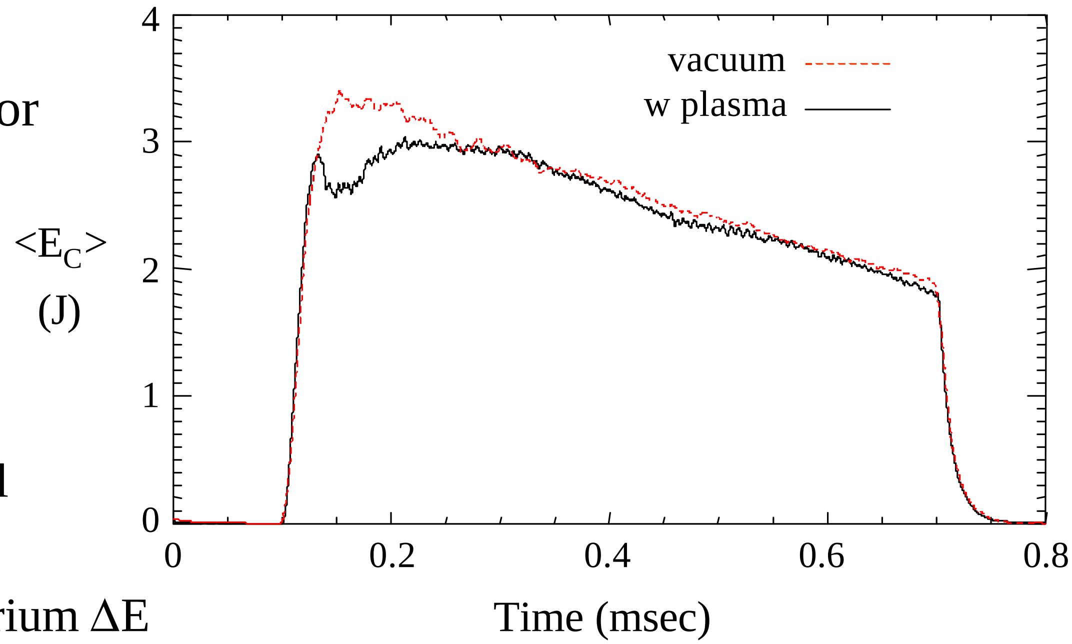
Maximum Energy Input to FRC Determined by Energy Loss from RMF Supply

Energy loss from RMF Capacitor Bank:

Vacuum Discharge $\rightarrow$ 1,470 J

Plasma Discharge $\rightarrow$ 1,520 J

50 J



To eliminate energy spent on initial ionization and radiation losses:

Vacuum ΔE - Discharge to equilibrium ΔE

\- 20 J

$\Rightarrow$ Max energy into FRC from 0.2 to 0.7 ms = 30 J

$$P_{max} \leq 60 \text{ kW} \pm 10 \text{ kW}$$

FRC Particle Confinement with RMF

Avg. Power lost from RMF antenna: $P_{RMF} \approx$ 40 kW

Ohmic power from FRC j_θ, j_z
(classical $\eta_\perp$ at kT_e = 33 eV) P_Ω = 40 kW

Power lost through ion equilibration:

$P_{ei} = 2.5\text{x}10^{-33}\ N \cdot n / T_e^{1/2}$ = 6 kW

Power lost due to impurity radiation:

$P_{rad} = 10^{-31}\ n_{imp} \cdot n_e \cdot Vol$ = 14 kW

Assume the remaining power loss is convective:

$\Rightarrow P_N = E_N\ dN/dt$ = 20 kW

During equilibrium phase (dashed lines):

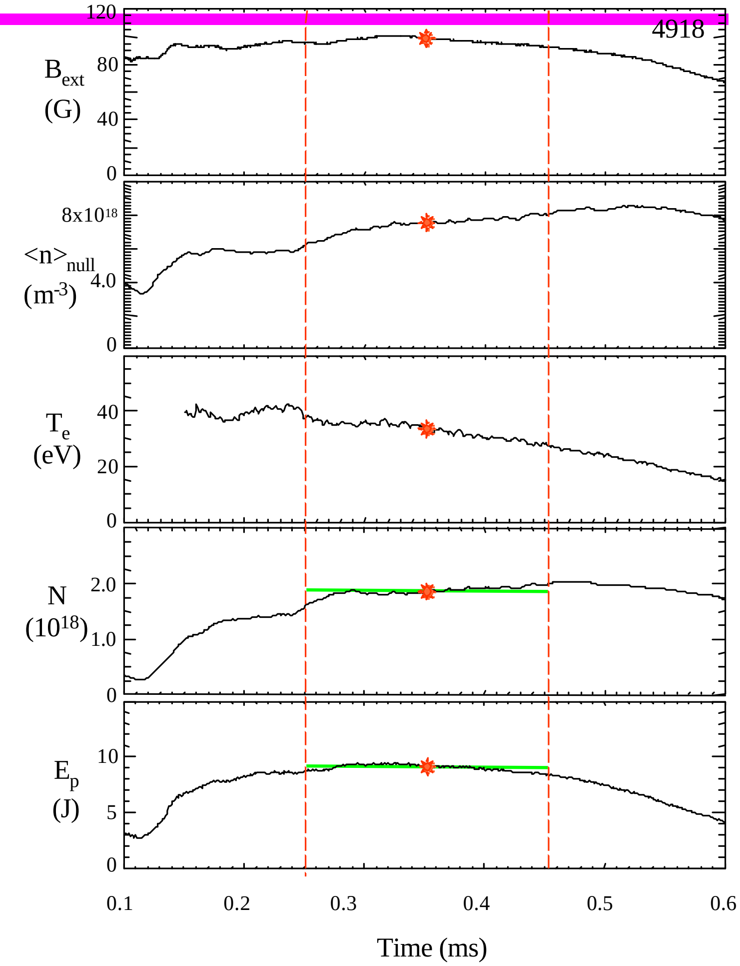
$$dN/dt = dE_p/dt = 0$$

(Loss per e^-) {Loss to ionize and heat}

$E_N = 5/2\ kT_e + \{5/2\ kT_e + E_{ionize}\} = 3\text{x}10^{-17}$ J

$\Rightarrow$ $\boxed{\tau_N \approx 2.7 \text{ ms}}$

Prior Maximum τ_N (LSX) ~ 1 ms



Impurity Line Radiation Power Loss (C and O)

For $15 \text{ eV} < kT_e < 50 \text{ eV}$
and $0 < t < 1 \text{ ms}$

$P_{rad} = 10^{-31}\, n_e\, n_{imp} \cdot Vol_{FRC}$

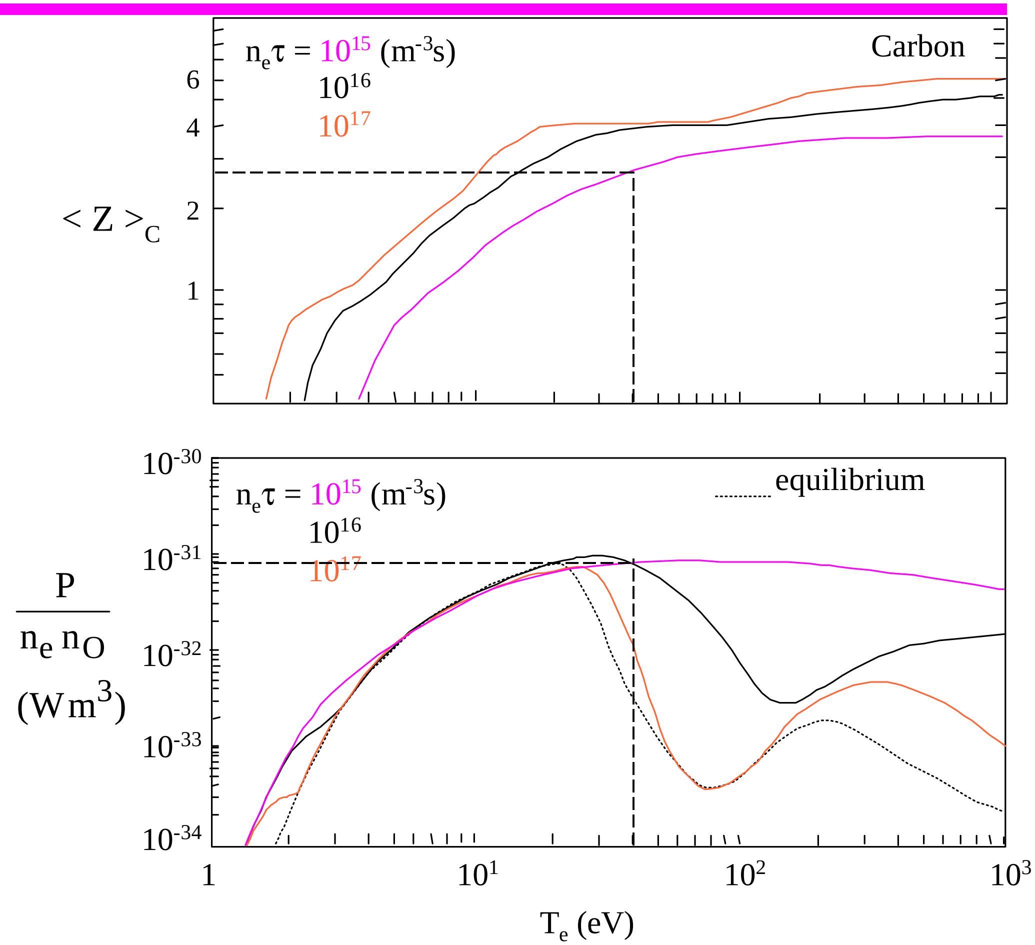
From CO_2 doping experiments:

$C \cong 0.5\%$, $O \cong 0.5\%$

$N = \langle n_e \rangle \cdot Vol_{FRC} = 1.8\text{x}10^{18}$

$n_{imp} = 0.01 \langle n_e \rangle = 8\text{x}10^{16} \text{ m}^{-3}$

$P_{rad} \geq 14 \text{ kW}$



Low $\eta_\perp$ Required For Reactor Efficiency with $B_\omega < 100G$

- Empirical flux lifetime scaling: $\tau_\varphi = r_s^2/16D_\perp = 40x_s^{1/2}r_s^2(10cm)n_m(10^{20}\ m^{-3})$.
- High Density Resistivity Scaling:

$$\boxed{D_\perp = \frac{\eta_\perp}{\mu_o} = \frac{5}{\sqrt{x_s n\left(10^{21}\,m^{-3}\right)}}\ m^2/s}$$

$$\frac{\gamma}{\lambda} = \sqrt{\left(\frac{F_\theta}{nm_e\omega r_s \nu_\perp}\right)_{r_s}} = \frac{\sqrt{2}B_\omega}{\mu_o n e r_s (D_\perp\omega)^{1/2}} = \frac{0.007B_\omega(G)}{n(10^{20})\omega^{1/2}(10^6)D_\perp^{1/2}r_s}$$

- Need $0.2ne\omega r_s^2 \approx B_e/\mu_o \Rightarrow$ $$\frac{\gamma}{\lambda} \approx \frac{0.013B_\omega(G)}{\sqrt{n(10^{20})D_\perp(m^2/s)B_e(T)}}$$

Better Resistivity Scaling Measured in Recent Low Density Experiments

$$\text{LSX Scaling}: D_{\perp} = \frac{\eta_{\perp}}{\mu_o} = \frac{15}{\sqrt{x_s n(10^{20} m^{-3})}} \ m^2/s$$

Device	r_c	r_s	B_e	j_p	T_t	n_m	$D_{\perp}$ (m^2/s)	
	(cm)	(cm)	(kG)	(mWb)	(eV)	(10^{20})	scaled	meas
LSX	45	14	8	4.5	1500	10	9	9
LSX	45	22	4	9.5	300	13	6	6
TCS	45	23	1.4	3.7	200	2.5	14	22*
TCS	45	18	1.4	1.8	350	1.4	20	10
FIX	40	16	0.4	0.4	100	0.4	38	11
STX	23	20	0.1	0.35	50	0.05	75	'30'

- Except for the higher density TCS* case, which is obviously influenced by impurities, the measured resistivity at low densities is at least a factor of two better than the LSX based (high density) empirical scaling.
- Considerable improvement is still needed for RMF to be efficient at 10^{20} m^{-3} densities.

Development Path

$$\frac{\gamma}{\lambda} = \frac{0.007\,B_\omega(G)}{n(10^{20})\omega^{1/2}(MHz)r_s(m)\sqrt{D_\perp(m^2/s)}}$$

Parameter	STX	STX/ug	TCS	POP	Reactor
R_c (m)	0.25	0.25	0.45	0.50	2.50
B_e (T)	0.01	0.03	0.10	0.3	1.25
n_e (10^{20} m^{-3})	0.05	0.15	0.50	1.0	2.0
T_e (keV)	0.05^	0.15^	0.25*	1.0*	10*
ω (10^6 s^{-1})	2	2	1	0.5	0.1
$B_e/\mu_o\omega n_e e(r_s^2/4)$ B_ω (G)	0.5 25	0.5 75	0.25 50-75	0.6 50	0.3 100
γ/λ	$12/\sqrt{D_\perp}$	$12/\sqrt{D_\perp}$	$2\text{-}3/\sqrt{D_\perp}$	$1.2/\sqrt{D_\perp}$	$0.6/\sqrt{D_\perp}$
ϕ (Wb) s	$0.35\text{x}10^{-3}$ 2	$1.0\text{x}10^{-3}$ 5.5	0.01 2.3	0.04 4.0	4 20

$*T_i = T_e$

$^\wedge T_i \approx 1\text{eV}$

- STX/upgrade will test ability to reach higher T_e as RMF power increases.
- TCS will test ability to achieve smaller $\eta_\perp$ with hot ions as size increases.
- POP device would investigate major physics questions in a TCS sized device.

Ion Spin-up

- Either neutral ion friction, ν_{in} or, equivalently, fueling, $\nu_f = s/n$ is required to reach a steady-state ion velocity $v_{i\theta} = v_{e\theta}/(1 + m_i\nu_f/m_e\,\nu_\perp)$
- $\nu_\perp = 3.5\text{x}10^6\, D_\perp(\text{m}^2/\text{s})$, so would require fueling rate of $10^3\ \text{sec}^{-1}$ if $D_\perp = 1\ \text{m}^2/\text{s}$ to prevent ions from spinning up to 1/2 electron speed. This is clearly impractical for a reactor.
- Two Solutions:
  - Central fueling at field null will provide outward v_r which can greatly reduce RMF power requirements, and thus RMF torque on electrons.
  - Neutral beams can be injected opposite RMF direction providing large source of oppositely directed angular momentum (since $v_{i\theta} << v_{ti}$).

Summary & Conclusions

- RMF current drive has been demonstrated to work for standard FRC with $B_\omega << B_z$. Well modeled by numerical calculations with synchronous electron rotation.
- RMF drive necessarily produces edge current which may be stabilizing influence.
- RMF frequency must be carefully chosen to match FRC parameters.
- Key parameter is effective resistivity which will determine required RMF strength and power. Central fueling could greatly reduce RMF power requirement and mitigate ion spin-up problem.
- Critical experiments will be carried out in the next few years using the STX and TCS facilities.